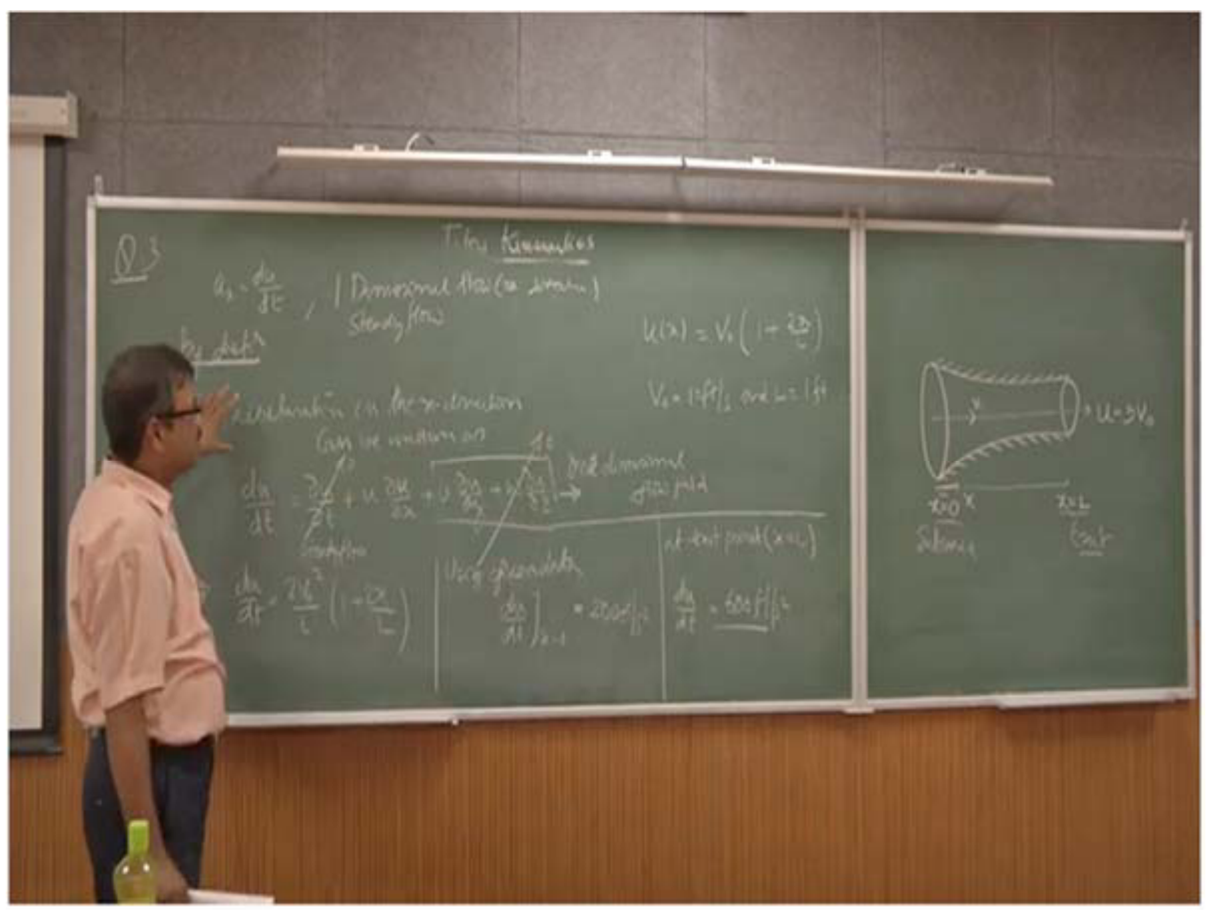

And at the entrance points and the velocity at the exit point will be the $3V_0$. And the velocity distributions is given with respect to the x directions this is the about the nozzles. The velocity field is given as the

$$u(x) = V_0\left[1 + \frac{2x}{L}\right]$$

This is what the velocity distributions given to us. What we have to need to compute it? What is the acceleration in the x directions? du/dt at the entrance point at this x = 0 that is what the entrance and this is what of exit of the flow.

So if it is the conditions we need to compute it what will be the accelerations ax du/dt.
If the $V_0 = 10 ft/sec$ and the length is 1 feet. Now if you look at these problems is very easy problems it may be has described as the converging nozzles giving the velocity increase of the velocity as the nozzle dimension is coming, decreasing trend. But in mathematically it is a very easy that we are defining it what could be the accelerations as the total derivative of u component with respect to the time.

And as given in the figure also I am just highlighting it this is the what 1 dimensional flow only x directions component what we have. That means the by definitions the accelerations and along the x directions accelerations the accelerations in the x directions can be written as very simple forms just taking the definitions that the accelerations will be total derivative of du/dt that is what will represent as a partial derivative of time, the local accelerations component and convective acceleration component in the x directions that what were look into it.

$$\frac{du}{dt} = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$$

$$\frac{\partial u}{\partial t} = 0 \rightarrow steady\ flow$$

$$v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = 0 \rightarrow one\ dimensional\ flow\ field$$

But if you look at these problems what are the components can be neglected as it is a steady flow there is no time component on this. We can make it this is equal to 0 and since it is a 1 dimensional flow as it is a steady flow when is no time component in the velocity field and it is a 1 dimensional flow. These the components also becomes 0 as it is 1 dimensional flow field. That is what also with these components all becomes 0.

So it is quite easy we have to compute this part. So if you just substitute this value and do a partial derivative of this u with respect to x and if you substitute it will get it the du that by substitute it is

$$\frac{du}{dt} = \frac{2V_0^2}{L}\left(1 - \frac{2x}{L}\right)$$

So it is very easy as you do a partial derivative with respect to x. If you do the partial derivative you can look it and substituting this the u value, you will get this value.

So we know this how this accelerations ax varies with respect to x and what is a functions in terms of V_0 and the L? So we have to find out what will be the velocity when x = 0 and x = L and the V_0 is given to us. So using given data we can compute it the accelerations in the x directions du/dt at x = 0. I am just substitute it in here when x = 0,

$$\left.\frac{du}{dt}\right]_{x=0} = 200\ ft/s^2$$

value substituting the V_0 and the L value that is what we will get it 200 feet per square.

The same way at exit point where x = L just substituting here, x = L value. We will get it du/dt = 600 feet per second square which is given is 3 times of V_0.

$$\frac{du}{dt} = 600\ ft/s^2$$

So if you look at that we can compute this the velocity is increasing, the accelerations also increasing than that point that is what we are computing using that substitute of it.

So let me summarize these problems one of the easy problems only it has described the velocity field giving a converging nozzles and telling this the velocity at this point is equal to V_0 and the velocity increases into 3 times of V_0 here and the velocity was described here and that what we are just substituting as computing the acceleration in the x directions only. That what we just substituting mathematically get it this value what will be the acceleration at the entrance point and the exit point. This is one of the easiest problems.

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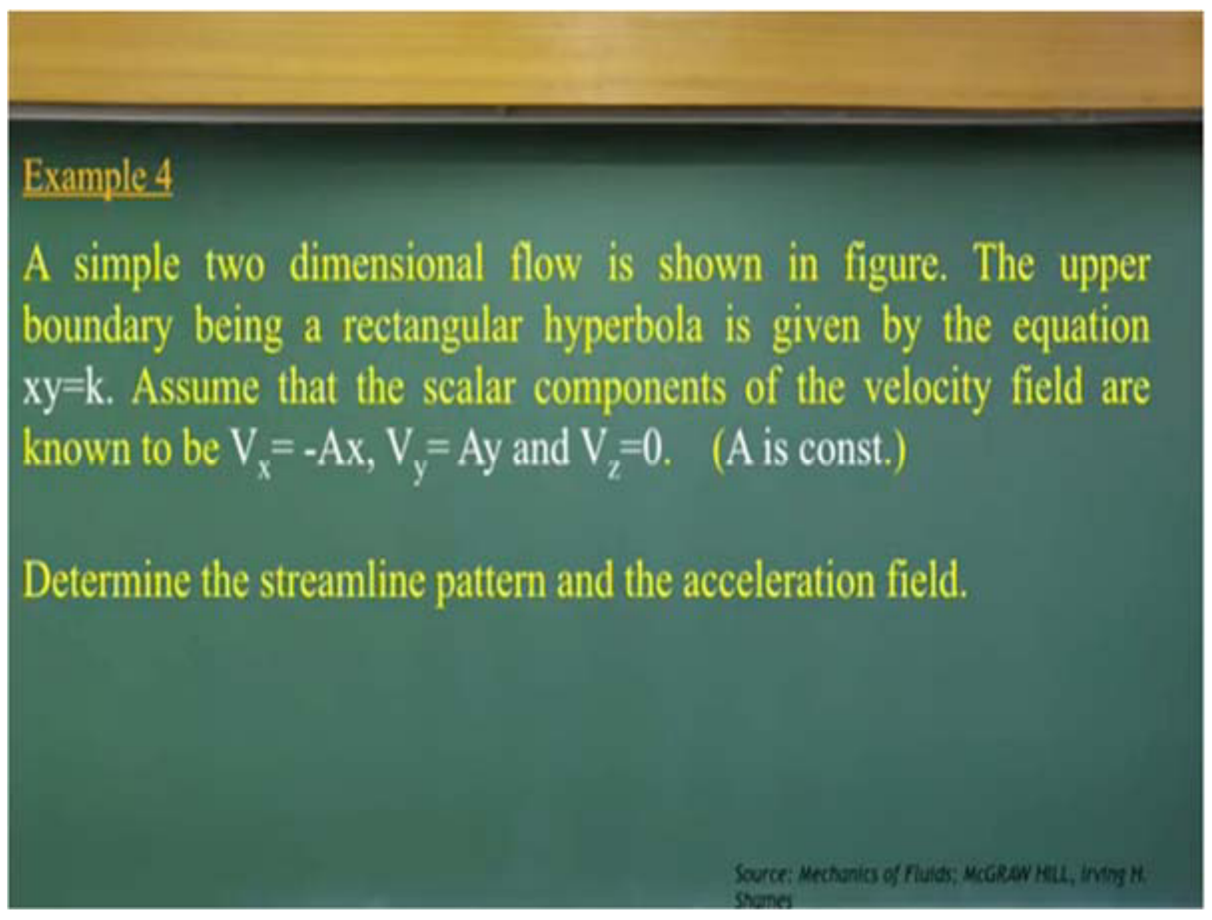

So let us come to the example 4 which is indicates that a simple 2 dimensional flow as shown in this figure.

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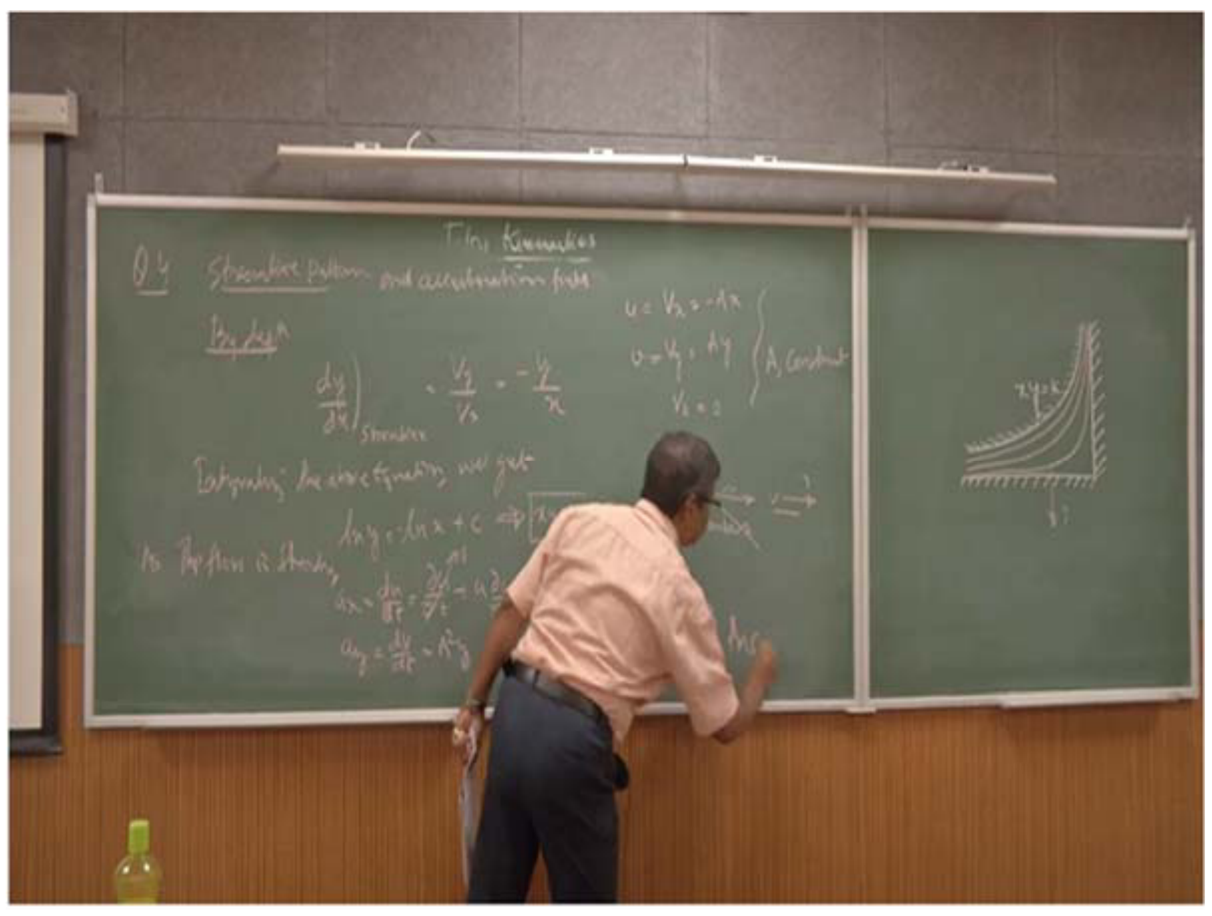

It is a 2 dimensional flow patterns, the upper boundary is being a rectangular hyperbola is given an equation is equal to $xy = k$

This upper part as given it that and assuming the scalar component of velocity fields are as given as the

$$u = V_x = -Ax$$
$$v = V_y = Ay$$
$$V_z = 0$$

as a 2 dimensional field and where the A is a constant. We have to determine the streamline pattern and also find the accelerations field.

The problems what we are looking at we have to determine now the streamline the pattern means we are looking at what could be the equations for that and also we are looking at what will be the accelerations field. This 2 the problems which is a very straightforward equation looking at that as we have given the velocity scalar components we have to find out the stream functions once you know the stream functions then we can derive what could be the accelerations field?

That means you can sketch the stream functions could be like this would come it as expanding it. So we can draw a streamlines like this. So we are looking at what is these functions? So defining this stream functions. Now if you look at the result part what we are looking at here that we need to put the basic equations of the streamline equations okay. The basic the definitions are the equations of the streamlines as the by definitions what we do it that

$$\left.\frac{dy}{dx}\right]_{streamline} = \frac{V_y}{V_x} = \frac{-y}{x}$$

this is what the definitions.

As you know it the when you take a tangential component of a streamlines that should be indicate in the velocity vector this is what the velocity vectors at this point and the tangential component are the parallel that is what the definitions of the streamlines. So we are defining that definitions in mathematically the slope of the streamline =Vy/Vx. So if I substitute this Vy/Vx you can just substituting these values we can get -y/x.

$$\frac{V_y}{V_x} = \frac{-y}{x}$$

So as we are looking at what could be the streamline functions. This is what we resize the streamline functions here see if I do integrations, the integrating the above equations we get that the value or we can write it very simple form is the

$$lny = lnx + C$$

$$xy = c$$

That means again it is indicating it also will have a hyperbolic nature of the streamlined functions. That is what is define the boundary the vectoral hyperbola functions where it is falling for the streamline functions that what we are getting from the is the equations for streamline.

Now if you look it to compute the accelerations components. So basically we look at 2 accelerations component as there is no the time components. We can as the flow is steady which is very easy to compute now. We can compute x values as given as earlier du/dt total derivative of y with respect to time. That is what we will substitute as in this case this is what the v, u as this is the v components.

$$a_x = \frac{du}{dt} = A^2x$$

$$a_y = \frac{dv}{dt} = A^2y$$

So these components become 0. So you know it is value to that but it does not have a functions of y and z. So these components 0 and this component 0. So substituting this value will get its value is equal to A^2x. Similar way we compute ay which is dv/dt writing the same expressions instead of u we can write a scalar component in the y direction is V and if I substitute it I will get it A^2y.

That means the accelerations becomes a is the accelerations field in the 2 dimensional that what will be the A^2x i, A^2y j component that is what the answer for these problem.

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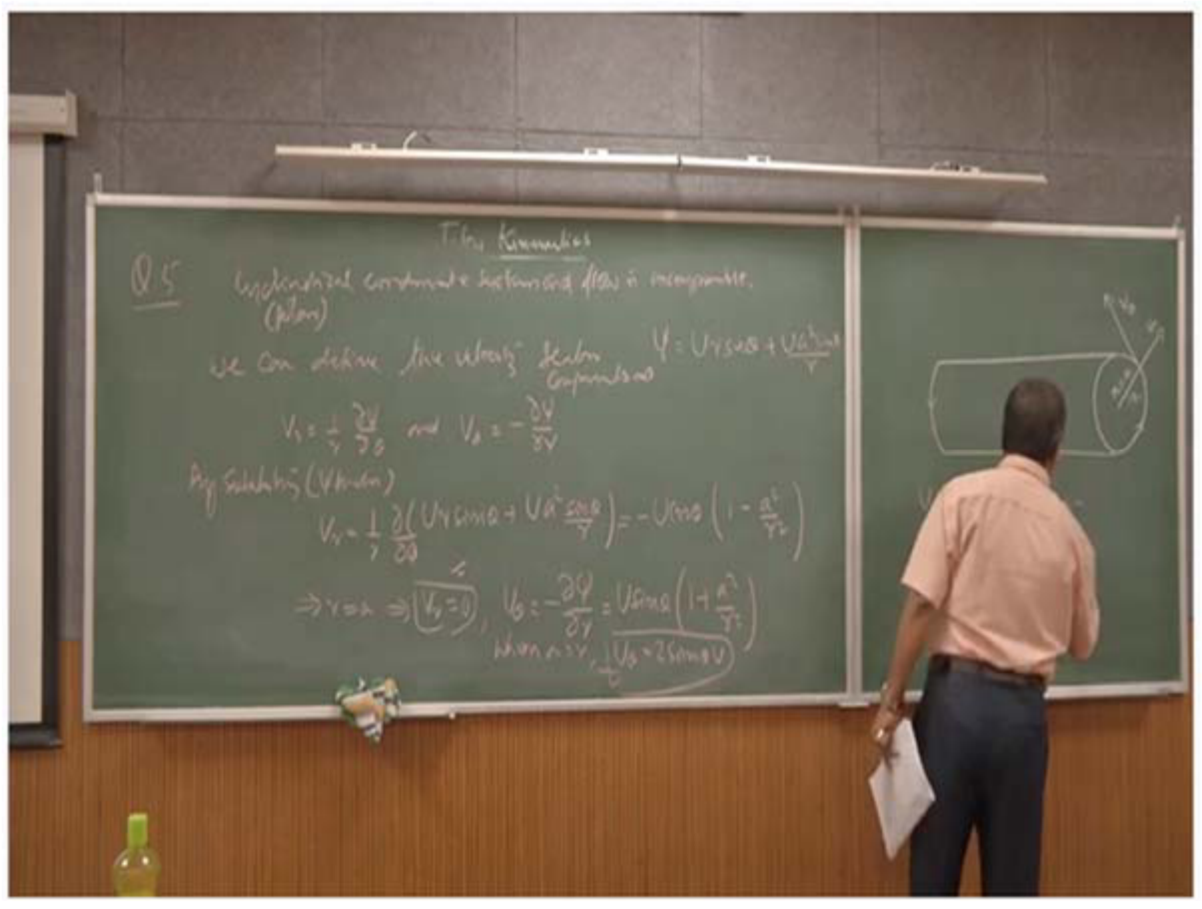

Let us solve the next examples that is example number 5, an incompressible flow around a circular cylinders of radius a as given in the figure is represented by a stream functions as given it here as

$$Ur\sin\theta + \frac{Ua^2\, sin\,\theta}{r}$$

r is the radius as a variables and the total radius of the cylinders is equal to the a, a is a radius of this and U is here represent as the free stream velocity.

So we need to compute it first we have to prove it the Vr the radial velocities is equal to 0 along the circles when r = y. So we have to first find out the radial velocity Vr is it equal to 0 when r = a value and the tangential V theta component also we have to find out the value of theta where the magnitudes of the velocity vectors should equal to the free stream velocity components. So it is bit analytical way to compute it.

But if you look it the stream function is given to us where to compute the radial velocity and the tangential velocity. So the problem is very easy but only the problem is here it has given in terms of cylindrical coordinate system. So we have cylindrical coordinate systems that is what we are using it. So we have to write this the velocity the function in terms of the stream function for a cylindrical coordinate systems as the flow is also the flow is incompressible okay.

That is a basic definitions here. So in cylindrical coordinates or the polar coordinate systems we can define it the velocity field we can define the velocity scalar component as given like this that

$$V_r = \frac{1}{r}\frac{\partial \Psi}{\partial \theta}$$

$$V_\theta = -\frac{\partial \Psi}{\partial r}$$

So the please refer to a books to know what is the definitions of the relationship between velocity scalar components and the stream functions for a polar coordinate system which is very same way what we do for that.

But just telling if you look it the 1/r component where it comes when you have partial derivation with respect to theta values. So as these problems is given to us we are just to find out whether the Vr the radial velocity component is equal to 0. So it is very easy things. Now let us compute it by getting substituting the Ψ functions. You can compute it that means I am substituting the stream functions has given it here is

$$Ur\sin\theta + \frac{Ua^2\, sin\,\theta}{r}$$

So if you substitute but do a partial derivatives with respect to θ and put it these value which really comes out to be

$$-U\cos\theta\left(1 - \frac{a^2}{r^2}\right)$$

If you look at these functions when the r becomes a, no doubt the V_r will be 0. Look at this function if substitute a = r this component become 0. So $V_r = 0$. That is what is the first component what we prove it. Now looking for the second component that what will be the value of V_θ?

$$r = a \Rightarrow V_r = 0$$

That means with respect to r just substituting this stream functions partial derivative with r and negative of that which indicates the tangential velocity component along the theta locations that what will come it here is

$$V_\theta = -\frac{\partial \Psi}{\partial r} = U \sin\theta \left(1 + \frac{a^2}{r^2}\right)$$

So when you have a becomes r that means the conditions when a = r value. So you get is

$$V_\theta = 2 \sin\theta \, V$$

That is the tangential velocity.

So r = a, the radial velocity 0 but the tangential velocity is equal to 2 Vsin θ Now you are looking it at what θ values we will have the resultant of the velocity vector is equal to the free stream velocity. This is quite easy for us that means we can as we know this V_r and V_θ value we can find out total magnitudes of the velocity vectors and that what we will equate to the 0 and that the conditions at which theta value its gives a value equal to the resultant velocity magnitude is equal to this is the conditions what we looking it.

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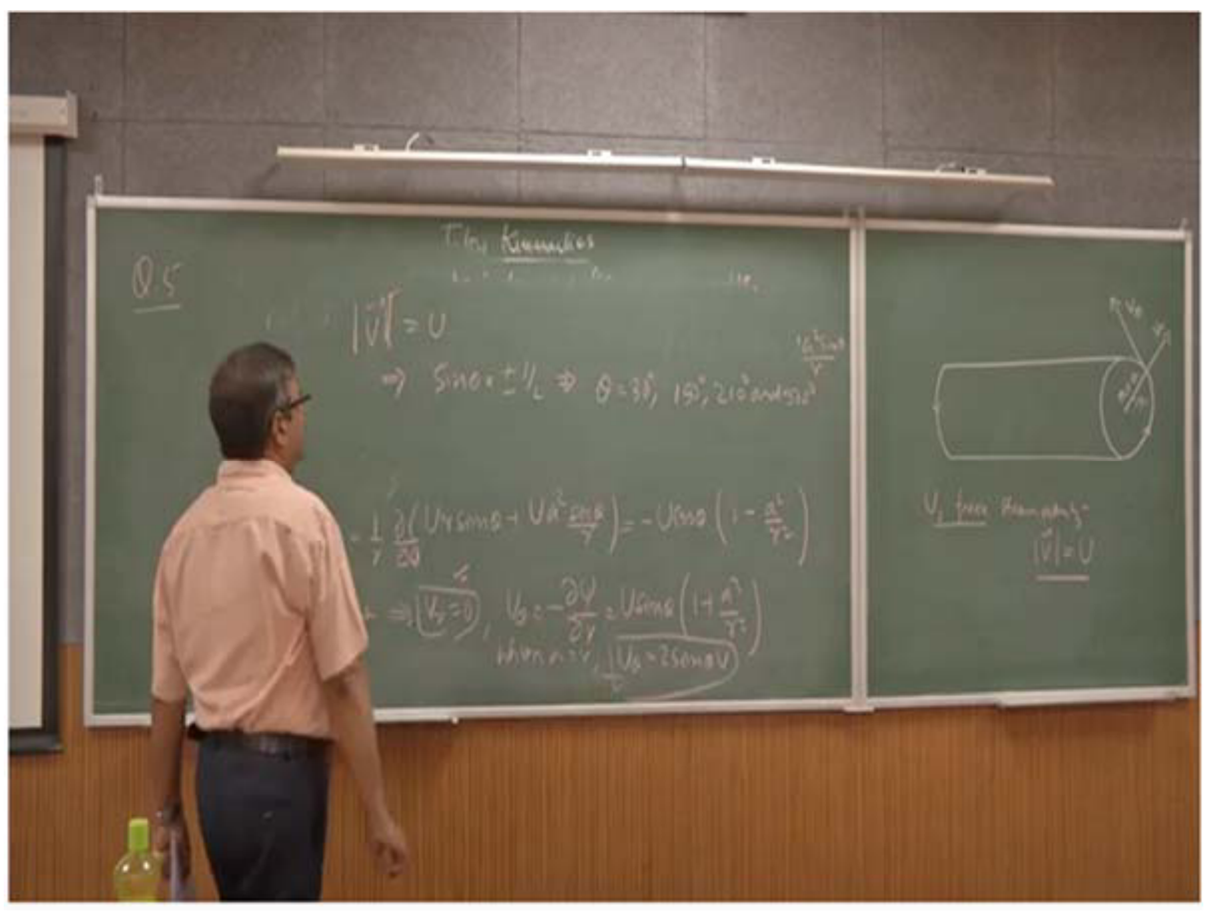

If I just as I know it V and V_θ and I can get the magnitudes and if I equate with U value, I will get

$$|\vec{V}| = U$$

$$\Rightarrow \sin\theta = \pm\frac{1}{2}$$

$$\Rightarrow \theta = 30^\circ, 150^\circ, 210^\circ, 330^\circ$$

So you can look it that you will have a $\theta = 30^\circ$ after that 150 degrees, 210 and 330 degrees you will have the magnitudes of your resultant velocity is will be equal to the free streams velocity.

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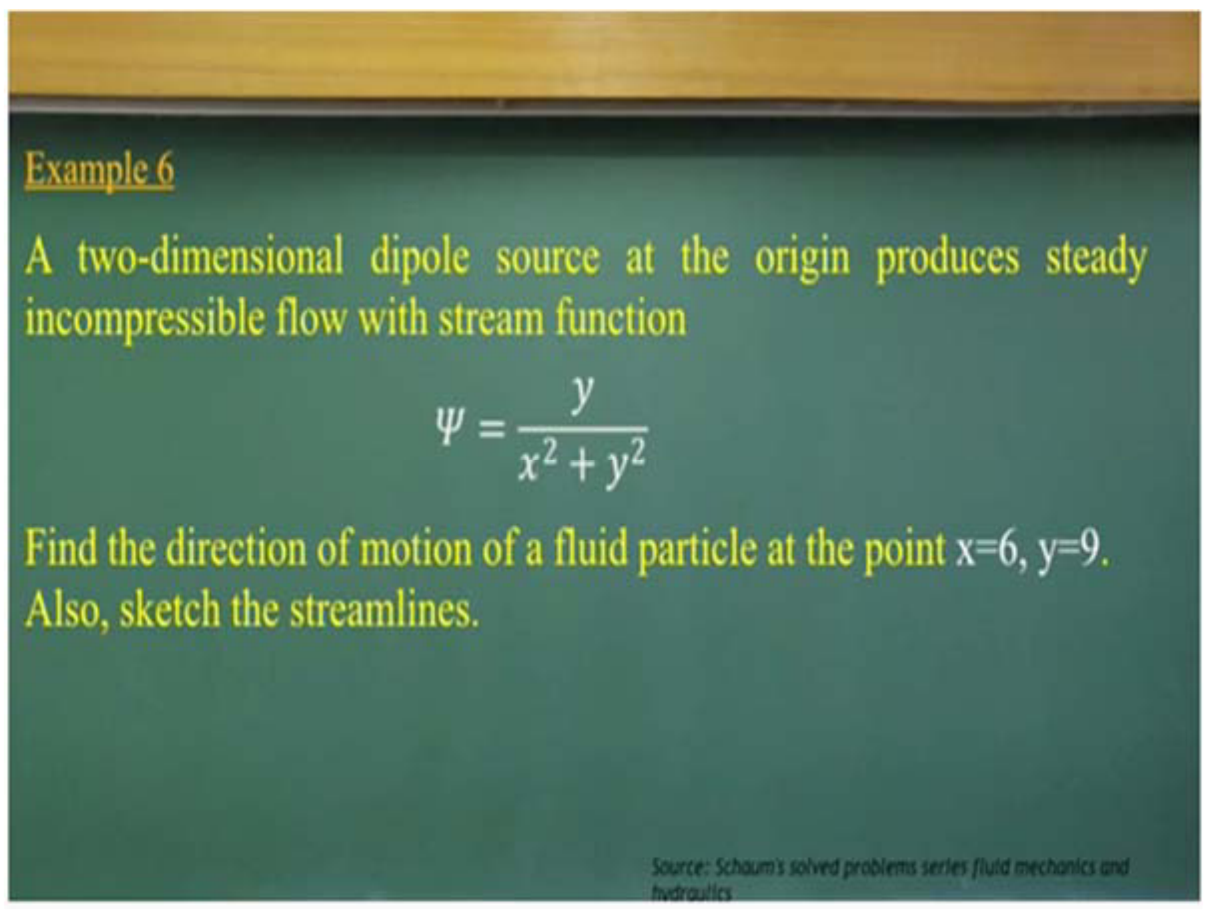

Which is very interesting the figures what you can see on the blackboard a 2 dimensional dipole source at the origins produces a steady incompressible flow with the stream functions as it is given.

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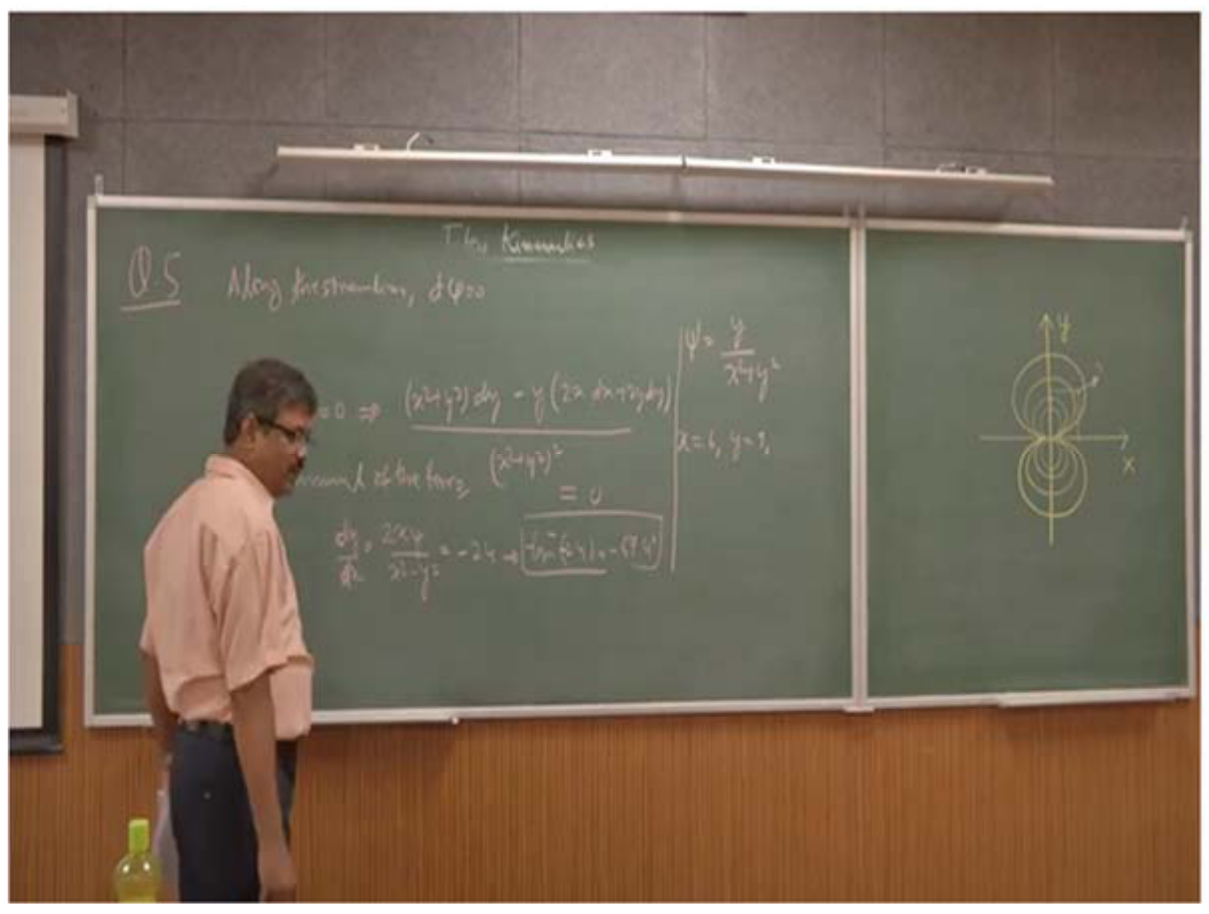

The stream functions is given to us we have to find out the directions of motion of the fluid particles at the x = 6 and y = 9. We need to know it what could be the directions of the fluid particles okay. So if we look at that, that means if it is a dipoles and the streamlines could be like these functions as you look at this function previous that means at a particular locations, we are looking it what could be the flow directions okay?

That means we have to find out if a streamlines are going through these what could be the functions of the streamline with respect to y and x, the slope components? If I know the slope component of that functions at that point that is what will give us the directions. So the first let us go for that we as know it that along the streamlines okay the $d\Psi = 0$ because along the streamline the stream functions value is constant. It does not wait.

So $d\Psi = 0$. That is what is indicate for us if I substitute the

$$d\Psi = 0$$

$$\Rightarrow \frac{(x^2 + y^2)dy - y(2xdy + 2ydy)}{(x^2 + y^2)^2} = 0$$

That is what is equal to 0. So here we have a very basic if a Ψ is a function of x and y, the total derivative of that. That is what we can expand it and substitute for that you can get it this components okay.

Just to follow any mathematics books to know it how it is expressions is coming it. If I rearrange this term then I can just rearrangement of the terms we can get

$$\frac{dy}{dx} = \frac{2xy}{x^2 - y^2} = -2.4$$

$$\Rightarrow \tan^{-1}(-2.4) = -67.4$$

So it is giving it that when you have a x and y value the velocity vectors will have an angle minus of 67.4 where you can find out what will be the directions. Now the second components were showing it that what could be the sketching of the streamlines.

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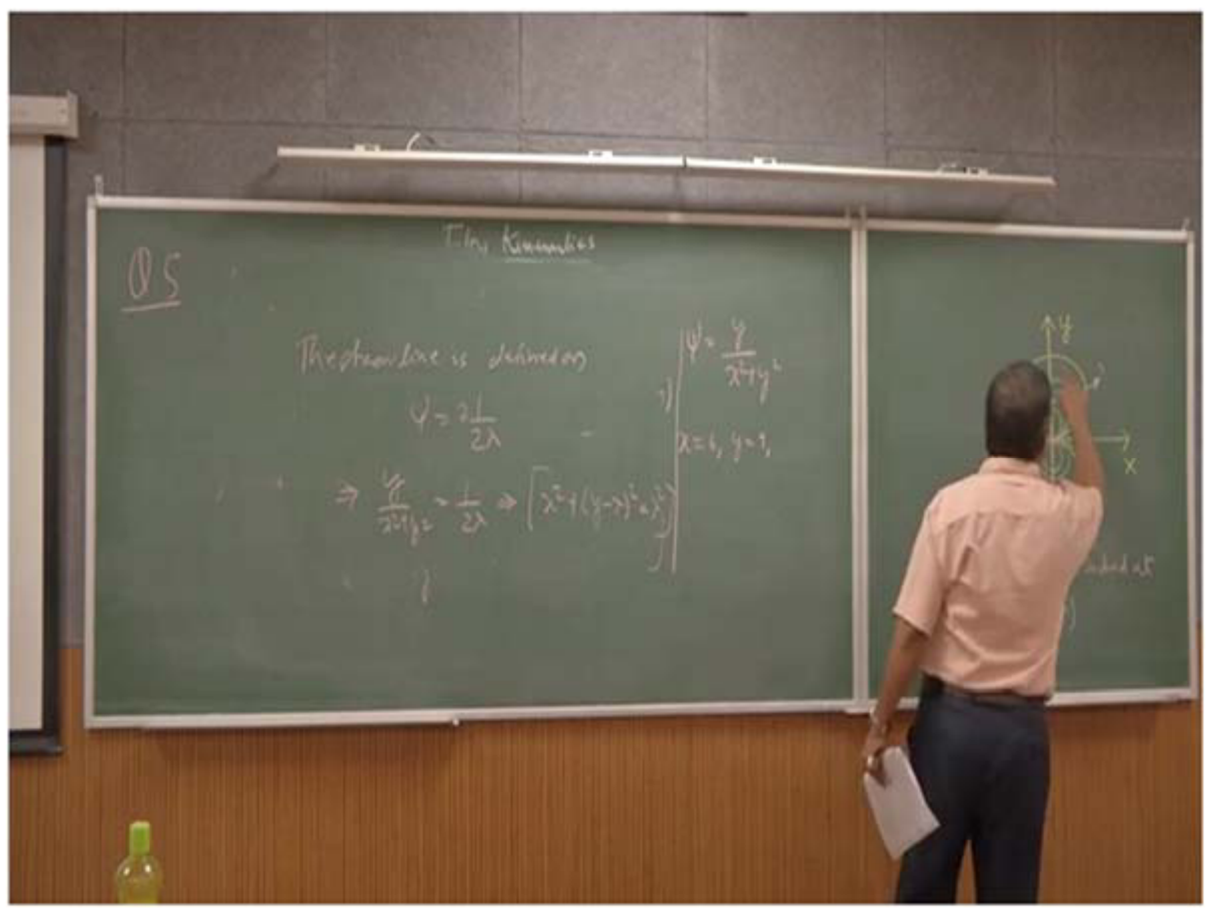

The sketching of the streamlines if you can find out that if you look in these streamline functions and since it is a dipole that means there is a 2 source components are there and we are drawing this streamlines patterns like this. So it is should be related into the functions in terms of a circular equations. So considering that part what we define it that the streamlines is defined as a constant is defined as

$$\Psi = 2\frac{1}{2\lambda}$$

2 lambda for our easy to write the circular equations for that.

That is the reasons 2 lambda is given. So what I am to say that before draw find out what could be the equations you should understand the flow things that what could be the conditions. In this case very easily can say that there will be the equation of the circles will be there to define these things. That is 2 reasons where define these the stream functions is 1/2 lambda values and if I substitute that value as

$$\frac{y}{x^2 + y^2} = \frac{1}{2\lambda}$$
$$\Rightarrow [x^2 + (y - \lambda)^2 = \lambda^2]$$

And these equations if I further simplified it which is the equations for a circles. We can write it like this form that is what is representing here the it is a circle of radius centred at 0 and the λ, as the λ will vary we will change it the circles different circles will get it. That is what is the solution for this. So with this let us complete this blackboard lectures. What we have prepared by solving the 6 problems. Thank you.